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VISUAL ART AND MATHEMATICS: THE MOEBIUS BAND

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Abstract–*The author discusses a topological form invented by A. F. Moebius and arrived at independently by the visual artist Max Bill. He describes briefly the mathematical properties of the Moebius band and its use in the works of several other artists. He points out that since the Renaissance few artists have been concerned with the relationships between mathematics and other manifestations of the human spirit.*

I.

Max Bill, Swiss architect, painter and sculptor, one of the pioneers of Concrete art, wrote the following in 1979: 'I created a single sided object by searching for a solution of a hanging sculpture turning in the rising air. The problem posed was to design a dynamic element for an electrical fireplace to be installed in a model building containing an exhibition of electrical appliances. My research was neither scientific nor mathematical, but purely *aesthetic*. This happened in 1935, and I named my sculpture "Endless Ribbon"' [1].

In 1936 Bill gave his ideas on Concrete art [2], which were an elaboration of those of Theo Van Doesburg (1930) [3]. Bill revised the 1936 text in 1949 [4] in which he said: 'We call "Concrete art" works of art which are created according to a technique and rules that are entirely appropriate to them, without taking external support from experimental nature or from its transformation, that is to say, without the intervention of a process of abstraction. Concrete art is autonomous in its specificity, it is the expression of the human spirit, destined for the human spirit, and should possess that clarity and that perfection which one expects from *works of the human spirit*.

'It is by means of Concrete painting and sculpture that those achievements which permit visual perception materialize. ... Abstract ideas which previously existed only in the mind are made visible in a concrete form.'

One of the 'abstract ideas' appears in Bill's sculpture 'Endless Ribbon', which was exhibited at the 1936 Triennial of Milan (Fig. 1). In 1935 he thought he was the first to discover the form that already had been described by the German mathematician August Ferdinand Moebius (1790–1860) in 1858. Also, in 1858, the German mathematician

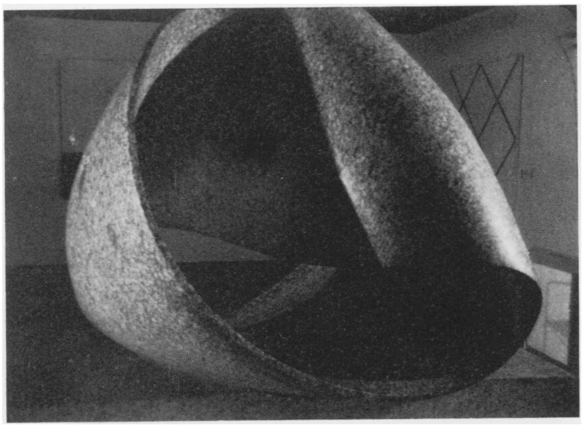

Fig. 1. Max Bill. 'Endless Ribbon', granite, 150 × 100 × 120 cm, 1935. (A second 'Endless Ribbon' was made in 1953.) (Centre National d'Art et de Culture Georges Pompidou, Paris)

J. B. Listing (1808–1882) published an article entitled *Census Rämlicher Complexe* in which a description of the construction of a one-sided surface appears [5]. In an interview with me, in French, in 1978 [6] Bill said: 'I made my first Moebius band in 1935 [out of paper] without being aware of the work of the mathematician Moebius.' In Ref. 7 he stated: 'I was fascinated by my new discovery; a knot with only one perimeter and only one continuous surface.' Also he said that his mathematical training in architecture was limited to calculus and that he had no particular interest in mathematics at that time.

The property of the Moebius band or strip [8], which Bill called an 'Endless Ribbon', is that it is possible to 'walk' on it, starting at any point P, and to arrive at a corresponding point on the 'other side' of the surface without penetrating the surface. Moreover, the surface has only one bounding edge. To clarify this property, consider the cylindrical surface shown in Fig. 2 (left). It has an exterior and an interior surface, and upper and lower edges. If one walks on the external surface starting at P, as indicated by the arrows, one cannot arrive at P on the internal surface without crossing an edge. If one

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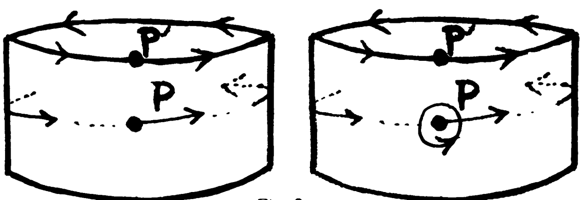

Fig. 2

starts at P′ on the upper edge of the cylinder, one cannot arrive at the lower edge without crossing the exterior or the interior surface. The Moebius band, however, makes it possible to make both of these 'walks'. As Bill realized, if you begin painting on one side of the band, you find that, when you have finished, you have completely painted the band.

An easy way to make a Moebius band with paper is the following, which was suggested by Moebius himself [8]: Start with a band of paper with corners marked A, B, A′, B′, as shown in Fig. 3, top. Fix side AB, twist the band around its horizontal axis so that the side A′B′ turns through 180° and then join sides AB and A′B′ (Fig. 3, bottom left).

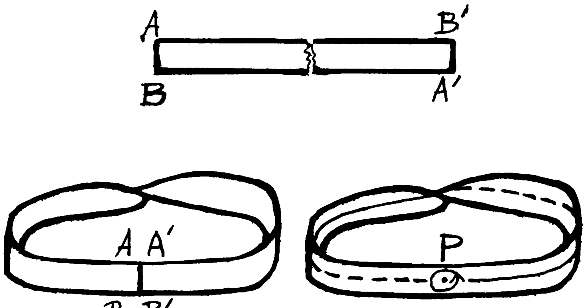

Fig. 3

The idea of the Moebius band stimulated the development of the branch of mathematics called topology; the term *topology* was introduced by Listing in his article [5]. The French mathematician H. Poincaré published in 1895 the first work that treated topology extensively [9].

Topology is concerned with the study of the properties of forms made of deformable materials that are maintained when the form undergoes deformations [10]. This involves the idea of orientation. To grasp what orientation means here, consider first Fig. 2 (right) that shows a cylindrical surface with point P on the surface surrounded by a circle. The arrow on the circle fixes its directional sense. If the point P and its circle make a complete tour along the external or the internal surface, as in Fig. 2 (right), and return to the starting point on the same surface, then the directional sense of the circle will remain unchanged and the cylindrical surface is said to be *orientable*. On the Moebius band, however, the directional sense of the circle around P reverses upon the completion of a tour along the band, because the point P and its circle at the end of the tour are on the other side of the surface to that on which the tour began. For this reason the Moebius band is said to be a *non-orientable* surface. Similarly, if the band from which a Moebius strip is formed is divided into an upper and lower half, point P starting from a location in the upper half will return after one tour in the lower half (Fig. 3, bottom, right) [10, pp. 269–270].

Bill has said that he was attracted to the Moebius band because (1) it has an endless surface that is finite and (2) it demonstrates the possibility of developing surfaces that lead to forms proclaiming the existence of aesthetic reality. But he doubts that topological forms could exist only by virtue of their aesthetic reality. He is sure that their effectiveness is due partly to their symbolic value, for example the Moebius strip resembles the mathematical symbol for infinity ∞ [11]. It is noteworthy that a mathematician and an artist, each on the basis of his own work, arrived at a form that has both mathematical and aesthetic properties.

II.

The Moebius band has several other mathematical properties [10, 12] not mentioned above, and artists continue to be fascinated with its aesthetic qualities, including those provided by color effects [13].

The Italian painter Corrado Cagli (1910–1976) on an Italian television program said that 'painting probably could not exist ... without the use and awareness of geometry and topology'. An example of Cagli's use of the Moebius band is shown in Fig. 4. A. Trombadori wrote in 1947: 'The way of Cagli

Fig. 4. *Corrado Cagli. Untitled, oil on canvas*, 50 × 40 cm, 1947. (Collection of Ebe Cagli Seidenberg)

was to denounce a world of painful symbols and realities through the spiritual and plastic logic of

metamorphosis, to overcome contradictions and to invent new harmonies. This poetic period of his activity was marked by a great interest in mathematical studies from which he had derived important reasons for elaborating structural and spatial themes in paintings' [14].

In a poem entitled 'To Corrado Cagli', Charles Olson of the U.S.A. [15] said, in part, the following: 'Upon a Moebius strip/Materials and weights of pain/their harmony.

A man within himself upon an empty ground/his head lay heavy on a huge right hand/itself a leopard on/his left and angled shoulder./His back a stave, his side a hole into the bosom of a sphere. ...' Evidently, the Moebius band or strip was a disquieting symbol also for Olson.

The Moebius strip also intrigued the Dutch artist M. C. Escher (1898–1972) (Fig. 5). He said that 'in

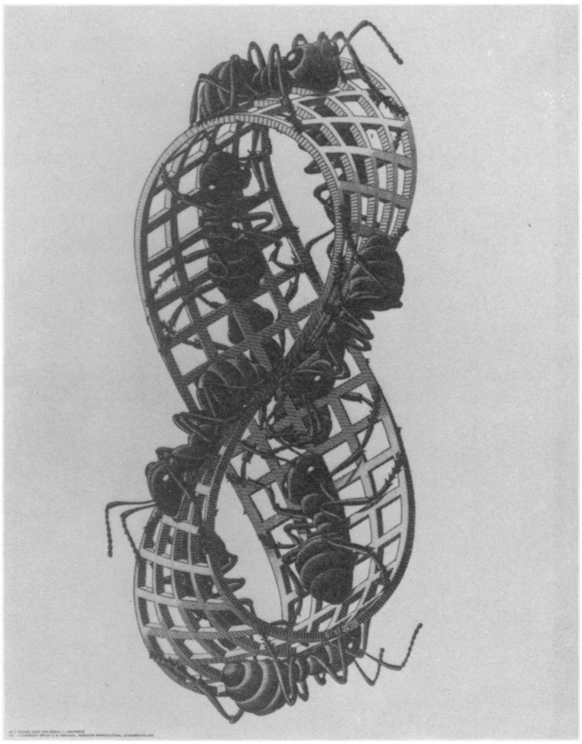

Fig. 5. M. C. Escher. 'Moebius Band, II', woodcut, 20 × 45 cm, 1963. (Collection of Gemeentemuseum, The Hague)

1960 an English mathematician encouraged me to make a print involving the Moebius band. I had scarcely heard it spoken about before' [16]. Bruno Ernst noted in his book [16], 'It is not necessary to understand Escher's statement too literally, because already in 1946 in his woodcut in three colors entitled "Cavaliers" he played with interesting topological forms that are analagous to the Moebius band'. Discussions of Escher's wide interest in aspects of mathematics will be found in Refs. 17 to 24. Escher said in 1961: 'By keenly confronting the enigmas that surround us and by considering and analyzing the observations that I made, I ended up in the domain of mathematics. Although I am absolutely (!) without training or knowledge in the exact sciences, I often seem to have more in common with mathematicians than with my fellow artists' [24].

In Fig. 6 is shown one of Escher's artistic pre-

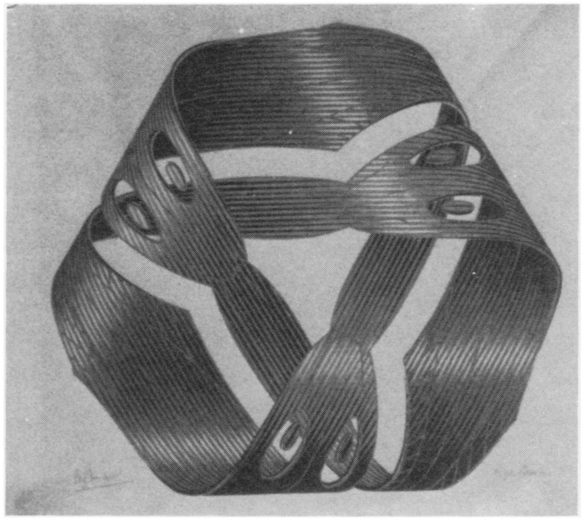

Fig. 6. M. C. Escher. 'Moebius Band, I', woodcut, 24 × 26 cm, 1961. (Collection of Gemeentemuseum, the Hague)

sentations of another strange property of the Moebius band. A anonymous poet wrote the following: 'A mathematician confided/that a Moebius strip is one-sided./You'll get quite a laugh/if you cut it in half,/for it stays in one piece when divided' [25].

Josè de Rivera of the U.S.A. has chosen, unconsciously, he told me, the form of the Moebius band for his sculpture 'Infinity' (Fig. 7). The German

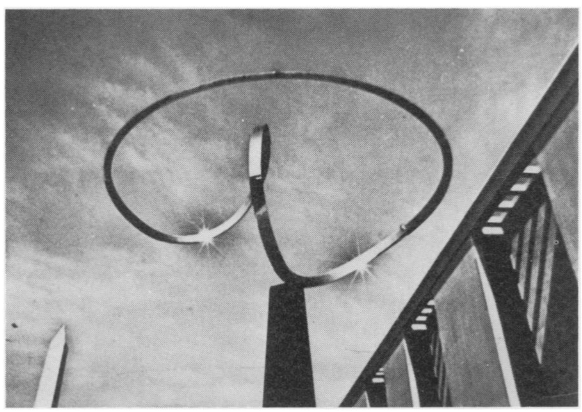

Fig. 7. Josè de Rivera. 'Infinity', 240 × 480 × 405 cm, 1972. (Collection of the National Museum of History and Technology, Washington, D.C.)

artist Jorg Neitzert has used the band as a symbol on an etching of an imaginary tomb of Moebius (Fig. 8) [26]. The Mexican artist Enrique Carbajol G. Sebastian makes transformable structures based on the Moebius band [27].

There are undoubtedly other artists who have been intrigued by the mathematical and aesthetic qualities of the band. The artists I have mentioned either appear to have had little training in mathematics and were unaware of the introduction of the form by Moebius or have chosen it for artistic symbols because of its enigmatic properties. It is no

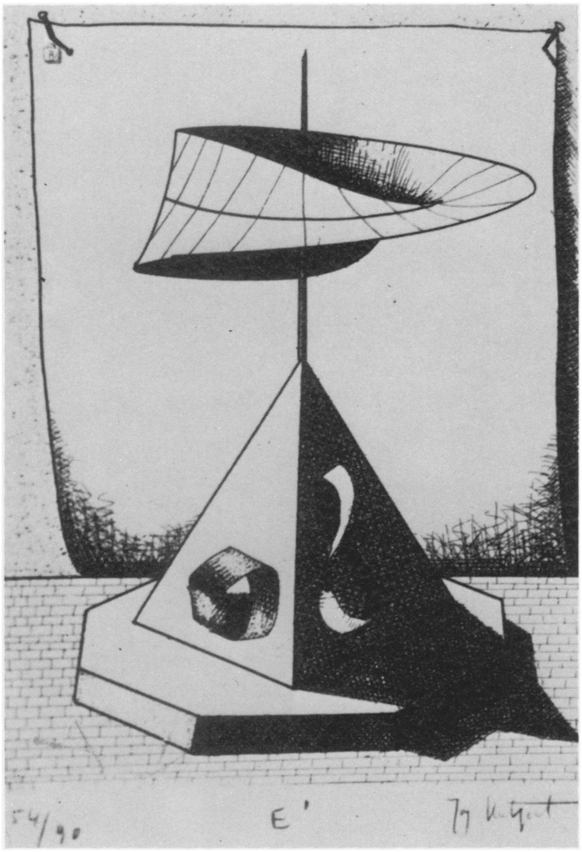

Fig. 8. Jorg Neitzert. 'C'Est tout tranquille où dorm Moebius', etching, 28 × 38 cm, 1973. (Reproduced from J. Neitzert, *Geometrie* (Paris: Atelier 25, 1973).)

doubt true that since the Rénaissance few artists have been concerned with the relationships between mathematics and other manifestations of the human spirit [28]. The purpose of my article is to draw attention to the way a simple topological form has been arrived at independently by a mathematician and by a visual artist and to how some artists have reacted to it by incorporating it in their works.

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